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HOMOGENEITY TESTING PROBLEMS IN BIRD STRIKE DATA PROCESSING WHEN SAMPLE SIZES ARE SMALL

Presented by Victor Y. Biryukov and Nikolai A. Nechval

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SUMMARY

This paper deals with testing the homogeneity of bird strike data when sample sizes are small. The traditional statistical approaches developed for large sample data processing will usually not be applicable in the above case. Using a method of conditioning on a sufficient statistic of the likelihood function of bird strike data, we develop some new homogeneity tests. The present paper undertakes a statistical analysis with respect to homogeneity testing problems in the Poisson and two-parameter exponential distributions. Tests are recommended on the basis of certain optimal power properties. The illustrative examples are given.

1. INTRODUCTION

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1. INTRODUCTION

Bird strike reporting systems have been in operation in many countries for a number of years and it is generally accepted that the statistics arising from these reports provide a valuable insight into the aviation bird hazard. However, bird strikes are comparatively rare events with the result that the data set available for statistical analysis is reasonably small. Consequently the traditional statistical approaches developed for large sample data processing will usually not be applicable in the above case.

The motivation for this paper is to focus attention on an exact statistical method which can be applied for bird strike data processing when sample sizes are small. This method is based on that certain conditional distributions, obtained by conditioning on a sufficient statistic, can be used to transform a data sample into a set of random variables whose distribution does not depend on the unknown (nuisance) parameters. The given distribution can be utilized in the solution which is strictly applicable when the parameters are completely specified.

2. A METHOD OF CONDITIONING ON A SUFFICIENT STATISTIC

This method consists in the following. Let $X^n=(X_1, \ldots, X_n)$ be a set of independent random variables that represent observations on a random variable X and are identically distributed with probability density function $f(x;\theta)$ indexed by an unknown (nuisance) parameter θ (in general, vector), $\theta \in \Theta^\circ$ (parameter space). Suppose that $T_n=t_n(X^n)$ is a sufficient statistic for θ with the probability density function $g(t_n;\theta)$. Let $v_{r+1}(x^n), \ldots, v_n(x^n)$ be new functions of x^n such that the transformation $x^n=(x_1, \ldots, x_n) \longrightarrow t_n=(t_{n1}, \ldots, t_{nr})$, $v=(v_{r+1}, \ldots, v_n)$ is one-to-one and smooth enough for the Jacobian to exist. Then the likelihood function of x^n can be transformed as

$$L(x^n;\theta) = f(t_n,v;\theta)\left|\frac{\partial(t_n,v)}{\partial x^n}\right| = g(t_n;\theta)f(v;t_n)\left|\frac{\partial(t_n,v)}{\partial x^n}\right|, \qquad (1)$$

where $f(t_n,v;\theta)$ is the joint probability density function of T_n and V,

$$f(v;t_n) = \frac{f(t_n,v;\theta)}{g(t_n;\theta)} = \frac{L(x^n(t_n,v);\theta)\left|\frac{\partial x^n}{\partial(t_n,v)}\right|}{g(t_n;\theta)} \qquad (2)$$

is the conditional probability density function of V given $T_n=t_n$ which does not depend on the unknown parameter θ (in virtue of the property of a sufficient statistic).

By relying upon the multivariate probability integral transformation of Rosenblatt (1952), an absolutely continuous density function $f(v;t_n)$ can be used to transform a set of random variables $X_1, \ldots, X_n$ into a smaller set of random variables that are identically and independently distributed with uniform distributions on the interval from zero to one.

To obtain a simple procedure for transforming a set of random variables $X_1, \dots ,X_n$ we need the following theorem.

Theorem 2.1. Let $X_1, \dots ,X_n$ be a sample of independent random variables that are identically distributed with probability density function $f(x;\theta)$ indexed by an unknown (nuisance) parameter θ (in general, vector), $\theta \in \Theta^\circ$ where Θ° is a given parametric set. Let $T_i=t_i(X_1, \dots ,X_i)$, $i=r(1)n$, be the sufficient statistics for θ such that $T_i=\underline{t}_i(T_{i+1},X_{i+1})$, $i=r(1)n-1$, and

$$g(\underline{t}_i(t_{i+1},x_{i+1});\theta)f(x_{i+1};\theta)\left|\frac{\partial \underline{t}_i}{\partial t_{i+1}}\right| = f(t_{i+1},x_{i+1};\theta)$$

$$= g(t_{i+1};\theta)f(x_{i+1};t_{i+1}), \tag{3}$$

where

$$f(x_{i+1};t_{i+1}) = \frac{g(\underline{t}_i(t_{i+1},x_{i+1});\theta)f(x_{i+1};\theta)\left|\frac{\partial \underline{t}_i}{\partial t_{i+1}}\right|}{g(t_{i+1};\theta)} \tag{4}$$

is the conditional probability density function of X_{i+1} given $T_{i+1}=t_{i+1}$. Then

$$\prod_{i=1}^{n} f(x_i;\theta)\left|\frac{\partial x^r}{\partial t_r}\right|\left|\prod_{i=r}^{n-1}\right|\frac{\partial \underline{t}_i}{\partial t_{i+1}}\left| \right| = \prod_{i=r+1}^{n} f(x_i;t_i)g(t_n;\theta). \tag{5}$$

Proof.

$$\prod_{i=1}^{n} f(x_i;\theta)\left|\frac{\partial x^r}{\partial t_r}\right|\left|\prod_{i=r}^{n-1}\right|\frac{\partial \underline{t}_i}{\partial t_{i+1}}\left| \right| = L(x^r;\theta)\prod_{i=r+1}^{n} f(x_i;\theta)\left|\frac{\partial x^r}{\partial t_r}\right|\left|\prod_{i=r}^{n-1}\right|\frac{\partial \underline{t}_i}{\partial t_{i+1}}\left| \right|$$

$$= \frac{L(x^r(t_r);\theta)\left|\frac{\partial x^r}{\partial t_r}\right|}{g(t_r;\theta)} \prod_{i=r}^{n-1} \frac{g(\underline{t}_i(t_{i+1},x_{i+1});\theta)f(x_{i+1};\theta)\left|\frac{\partial \underline{t}_i}{\partial t_{i+1}}\right|}{g(t_{i+1};\theta)}$$

$$\cdot\, g(t_n;\theta) = \prod_{i=r+1}^{n} f(x_i;t_i)g(t_n;\theta), \tag{6}$$

where r is the minimum size of sample required for constructing a sufficient statistic T_r for θ.

Corollary 2.1.1. The procedure of transforming $X^n=(X_1, \dots ,X_n)$ is defined by

$$\prod_{i=1}^{n} f(x_i;\theta) \longrightarrow \prod_{i=r+1}^{n} f(x_i;t_i). \tag{7}$$

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3. AN ITERATED PROCEDURE FOR TESTING THE HOMOGENEITY

Let $X_1, \ldots, X_n$ be a random sample of n observations on a random variable X with cumulative distribution function F(x) and probability density function f(x). The general problem of testing the homogeneity of n observations consists in testing the null hypothesis H_0 that $F(x)=F(x;\theta_1, \ldots, \theta_s)$ for every x_i, i=1(1)n, against unspecified alternatives. Here the θ_q's denote the parameters of the hypothesized cumulative distribution function F_0. In the sequel $f_0(x;\theta_1, \ldots, \theta_s)$ will denote the hypothesized probability density function. Two cases of this problem are of interest.

Case 1. The hypothesis H_0 is simple, that is the cumulative distribution function F_0 under H_0 is completely specified as to its functional form, such as normality, exponentiality, etc., as well as to the values of the parameters θ_q involved.

Case 2. The hypothesis H_0 is composite. This is when only the functional form of F_0 is given, and some or all of the parametric values are left unspecified.

The given paper deals with Case 2, that is we assume that under H_0 only the functional form of F_0 is given to us but one or more of the parameters θ_q ($q=1, \ldots, s$) are left unspecified. This case is in fact of more relevance since situations are very rare in practice where the cumulative distribution function to be tested is completely specified. It is well known that the classical Pearson chi-squared test of goodness-of-fit (χ^2) can be modified to fit this case by properly estimating the unspecified parameters. But it possesses an element of arbitrariness in the choice of group boundaries and one of the objections to this procedure is that its validity is questionable when the sample size n is small, i.e., it does not have the desirable property of being an exact test in the sense of giving exact probabilities of rejection when the null hypothesis is true.

In this paper we propose an iterated procedure (free from the above objections) for testing the null hypothesis H_0 that n independent observations $X_1, \ldots, X_n$ come from a common specified distribution with a common but unspecified parameter, i.e., in other words, for testing the homogeneity of n observations.

The proposed procedure is based on the above method of conditioning on a sufficient statistic and consists in that we consider the test of the null hypothesis H_0 of the homogeneity of $n \geq 2$ independent observations. We resolve this hypothesis into the following sequence of nested hypotheses: $H_0(2)$, the homogeneity of the first two observations; $H_0(3)$, the homogeneity of the first three observations; ... ; $H_0(n)$, the homogeneity of all n observations. Here, the test of $H_0(j)$ is not made unless $H_0(j-1)$ is accepted, $j=3,4, \ldots, n$. Then the homogeneity of the n observations is accepted if and only if all of $H_0(2), H_0(3), \ldots, H_0(n)$ are accepted. Let $W_2, W_3, \ldots, W_n$ denote, respectively, the n-1 test statistics. These test statistics are so selected that they are not only mutually stochastically independent but each W_j is a function of the first j observations alone, $j=2,3, \ldots, n$. This last condition is extremely important to our procedure because if $H_0(j)$ is rejected, using W_j, $j<n$, we stop the testing at that point; thus there is no need to perform the test for the last n-j

observations. For example, suppose $H_0(2)$, and thus $H_0(n) \equiv H_0$, is rejected; we then are not required to go to the trouble and expense of performing the test for the third through n observations since we do not need to compute the statistics $W_3, \ldots, W_n$.

Let a_j be the significance level of the test of $H_0(j)$, $j=2,3,\ldots,n$. The mutual independence of $W_2, W_3, \ldots, W_n$ implies that the significance level of the test of the homogeneity of the n observations by this iterated scheme is

$$a = 1 - \prod_{j=2}^{n}(1-a_j). \tag{8}$$

It is important to emphasize at this point that this probability is the significance level of this overall test even though the sequence of tests is truncated with the test of $H_0(j)$, $j<n$. For example, if $H_0(2)$ is rejected, we have that H_0 is rejected at significance level

$$a = 1 - \prod_{j=2}^{n}(1-a_j), \tag{9}$$

not simply a_2. Moreover, we have some reason as to why all n observations are not homogeneous; namely, it seems that the first two observations are not homogeneous. Now at this point, it is quite possible that the experimenter would desire to formulate a new hypothesis, such as the homogeneity of the last n-1 observations. This can then be tested in the manner outlined above with n replaced by n-1.

We illustrate this procedure with two important applications.

4. HOMOGENEITY TESTING FOR THE POISSON PROCESS

Let $X(u)$, $u \geq 0$, be the Poisson process with probability mass function

$$f(X(u)=x;b) = \frac{(bu)^x}{x!}e^{-bu} \quad (b>0,\ x\geq 0), \tag{10}$$

where $X(u)$ represents the number of events occurring in the interval $(0,u)$, $X(0)=0$ with probability 1, b is the rate parameter.

To introduce the Poisson homogeneity testing problem, we suppose that we observe n independent random variables $X_1(u_1), \ldots, X_n(u_n)$. Under the null hypothesis of homogeneity, each $X_i(u_i)$ follows a distribution (10) governed by the same parameter b, i.e.,

$$f_0(X_i(u_i)=x_i;b) = \frac{(bu_i)^{x_i}}{x_i!}e^{-bu_i}, \ \forall i. \tag{11}$$

In some problems b may be known, in others unknown. The u_i's, however, denote constants which are always given rather than unknown. For example, if $X_i(u_i)$ is the number of bird strike incidents incurred during ith of n intervals then u_i could be the number of aircraft movements (in terms of 10,000 movements)(or flying hours) and b the average bird strike incident rate per 10,000 aircraft movements (or flying hours).

We wish to test the null hypothesis (11) against an alternative hypothesis of non-homogeneity. By non-homogeneity we mean that, roughly speaking, the $X_i(u_i)$'s are more "spread out" than under the null hypothesis, either as a result of b being different for different i or else as a result of some kind of non-independence of events.

An iterated procedure for testing the homogeneity of n observations from the Poisson process (for the case of unknown b) is based on the transformation

$$\prod_{i=1}^{n} f_0(X_i(u_i)=x_i;b) \longrightarrow \prod_{i=2}^{n} f(x_i;t_i,p_i), \tag{12}$$

where

$$f(x_i;t_i,p_i) = \binom{t_i}{x_i} p_i^{x_i}(1-p_i)^{t_i-x_i}, \quad 0 \leq x_i \leq t_i, \tag{13}$$

$$t_i = \sum_{q=1}^{i} x_q , \tag{14}$$

$$p_i = u_i \Big/ \sum_{q=1}^{i} u_q . \tag{15}$$

Here, at ith stage, the hypothesis $H_0(i)$, $i \in \{2, \ldots, n\}$, is accepted if

$$x_i \in [x_L, x_U], \tag{16}$$

where x_L and x_U satisfy the relations

$$\begin{cases} \sum_{x_i=0}^{x_L-1} f(x_i;t_i,p_i) \leq a_i/2 \\ \sum_{x_i=0}^{x_L} f(x_i;t_i,p_i) > a_i/2, \end{cases} \tag{17}$$

and

$$\begin{cases} \sum_{x_i=x_U+1}^{t_i} f(x_i;t_i,p_i) \leq a_i/2 \\ \sum_{x_i=x_U}^{t_i} f(x_i;t_i,p_i) > a_i/2, \end{cases} \tag{18}$$

respectively.

Note that the sample range (when the sample size n is small) is also useful in testing the homogeneity of n observations from a common Poisson process (10), since it is known that the conditional distribution of n observations x_i from (10) subject to

$$t_n = \sum_{i=1}^{n} x_i = \text{constant}$$

is the multinomial,

$$f(x_1, \ldots, x_n; t_n, p_1, \ldots, p_n) = \frac{t_n!}{x_1! \ldots x_n!} p_1^{x_1} \ldots p_n^{x_n}, \quad (19)$$

where

$$p_i = u_i / \sum_{q=1}^{n} u_q, \quad i=1(1)n. \quad (20)$$

The exact distribution of the range $\underline{r}$ conditional upon t_n=constant can be computed from (19) for a variety of n and nominal levels of significance a, giving values $\underline{r}_a$ such that

$$\Pr(\underline{r} \geq \underline{r}_a) = \sum_{\underline{r} \geq \underline{r}_a} f(x_1, \ldots, x_n; t_n, p_1, \ldots, p_n) \leq a. \quad (21)$$

For the sake of illustration, let us suppose that we observe two independent random variables $X_1(u_1)=6$ and $X_2(u_2)=0$, where $u_1=u_2=3$, with the range $\underline{r}=6$. It follows from (19) that, for n=2 and $t_2=6$, $\underline{r}=6$ is significant at the probability level equal to 0.03125. If the nominal level of significance a=0.05, there is evidence against the assumption of a common Poisson process. Note that the same result, in this case, can be obtained by the iterated procedure.

The test based on the Poisson range supplements the usual index of dispersion of equation

$$\chi^2_{n-1} = \sum_{i=1}^{n} \frac{[X_i(u_i) - u_i \hat{\delta}]^2}{u_i \hat{\delta}}, \quad (22)$$

where

$$\hat{\delta} = T_n / \sum_{i=1}^{n} u_i \quad (23)$$

and

$$T_n = \sum_{i=1}^{n} X_i(u_i), \quad (24)$$

which is approximately distributed as χ^2 with n-1 degrees of freedom (see, e.g. Rao, 1952, pp. 205-6).

Table 1 (see below) includes the bird strike data taken from Thorpe and Wessum (1982). Using the statistic (22) for testing the homogeneity of 4 observations from Table 1 for the Poisson process (10) we obtain X^2_3=4.975. From tabulations of the statistic χ^2_3 we get $\Pr(\chi^2_3 \geq 7.81)=0.05$. Since our computed X^2_3 is smaller than 7.81 (at the nominal level of significance a= 0.05) we con-

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3. France
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clude that there is no evidence against the assumption of a common Poisson process with the common parameter b, i.e.,

$$X_i(u_i) \sim \frac{(\hat{b}u_i)^{x_i}}{x_i!} e^{-\hat{b}u_i}, \quad i=1(1)4, \tag{25}$$

where $\hat{b}=3.2656931$ represents the maximum likelihood estimate of b.

TABLE 1. National Reporting - 1980

Country	Number of bird strike incidents	Number of aircraft movements (in terms of 10,000 movements)
i	x_i	u_i
1. Austria	21	7.0000
2. Denmark	51	15.6463
3. France	134	47.7637
4. United Kingdom	356	101.6821

5. HOMOGENEITY TESTING FOR THE EXPONENTIAL DISTRIBUTION

Considerable attention has been given in the literature to the problem of testing a composite null hypothesis H_0 that a set of variables $Y_1, \ldots, Y_n$ represents a set of independent identically distributed exponential random variables with probability density function

$$f(y;\theta) = (1/\theta)\exp(-y/\theta), \quad y \geq 0, \tag{26}$$

for some unspecified common parameter $\theta > 0$. Some authors discuss the hypothesis that the variables have a common two-parameter exponential distribution with probability density function

$$f(y;\grave{a},\theta) = (1/\theta)\exp(-(y-\grave{a})/\theta), \quad y \geq \grave{a}, \tag{27}$$

for unspecified $\grave{a} \in (-\infty, \infty)$ and $\theta > 0$.

For testing the homogeneity of n observations from a common exponential distribution, a procedure is used that involves transformation of the data resulting in (n-2) new variables, which under the null hypothesis of homogeneity are distributed as independent observations from the uniform distribution on [0,1]. Thus, the many known tests of this completely specified distribution can be used.

Let $Y_1, \ldots, Y_n$ be a random sample of size n from a two-parameter exponential distribution (27). It will be convenient throughout the paper to denote the order statistics of $Y_1, \ldots, Y_n$ by $Y_{(1)} \leq Y_{(2)} \leq \cdots \leq Y_{(n)}$. Then

$$f(y_{(1)}, \ldots, y_{(n)};\grave{a},\theta) = n!\prod_{i=1}^{n} f(y_{(i)};\grave{a},\theta) \longrightarrow \prod_{i=2}^{n} f(z_i;\theta), \tag{28}$$

where

$$f(z_i;\theta) = (1/\theta)\exp(-z_i/\theta), \quad z_i \geq 0, \tag{29}$$

$$z_i = (n-i+1)(y_{(i)}-y_{(i-1)}), \ i=2(1)n. \tag{30}$$

Using the method of conditioning on a sufficient statistic, we have

$$\prod_{i=2}^{n} f(z_i;\theta) \longrightarrow \prod_{i=3}^{n} f(z_i;t_i), \tag{31}$$

where

$$f(z_i;t_i) = \frac{i-2}{t_i}\left[1 - z_i/t_i\right]^{i-3}, \quad 0 \leq z_i \leq t_i, \tag{32}$$

$$t_i = \sum_{q=2}^{i} z_q, \ i=3(1)n. \tag{33}$$

Now if the probability integral transformation defined by

$$w_i = F(z_i;t_i) = \int_0^{z_i} f(\check{z}_i;t_i)\,d\check{z}_i = 1 - (1 - z_i/t_i)^{i-2}, \quad i=3(1)n, \tag{34}$$

is used, we obtain

$$\Pr(W_i \leq w_i; i=3(1)n) = \int \cdots \int_{\{\check{z}_i: F(\check{z}_i;t_i) \leq w_i; i=3(1)n\}} \prod_{i=3}^{n} dF(\check{z}_i;t_i)$$

$$= \int_0^{w_n} \cdots \int_0^{w_3} \prod_{i=3}^{n} dw_i = \prod_{i=3}^{n} w_i, \tag{35}$$

where $0 \leq w_i \leq 1$, $i=3(1)n$. Hence $W_3, \ldots, W_n$ are uniformly and independently distributed on $[0,1]$ random variables.

To test the null hypothesis H_0 we can use, for example, K. Pearson's probability product test

$$P_{n-2} = -\ln\prod_{i=3}^{n} W_i, \tag{36}$$

a $\Gamma(n-2,0,1)$ random variable, or, equivalently,

$$2P_{n-2} = -2\ln\prod_{i=3}^{n} W_i, \tag{37}$$

a $\chi^2_{2(n-2)}$ random variable.

To illustrate the above procedure for executing a test of the ho-

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TABLE 2. Nationa

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Belgium
Denmark
Federal Republic of Germany
Finland
France
United Kingdom

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mogeneity for the exponentiality, the following data were taken from the paper of H. Dahl (1982) (see Table 2).

TABLE 2. National Reporting - 1982

Country	Airports	Costs per average yearly bird strike ($)
	i	y_i
Austria	1. Vienna	2,400
Belgium	2. Brussels	1,800
Denmark	3. 12 airports	3,200
Federal Republic of Germany	4. Civil airports	8,000
Finland	5. Helsinki-Vantaa	1,700
France	6. Lyon	2,350
	7. Charles de Gaulle	7,400
	8. Orly	2,500
	9. Marseille	9,000
	10. Nice	2,000
United Kingdom	11. Military airfields	7,000

Applying (37) we have

$$2P_9 = -2\ln\prod_{i=3}^{11} W_i = 15.12. \qquad (38)$$

$2P_9$ is a χ^2_{18} random variable iff the Y_i are drawn from a common two-parameter exponential distribution. At the 5% level of significance, we get from χ^2-tables that $Pr(\chi^2_{18} < 28.87) = 0.95$. Since the observed value 15.12 of our χ^2_{18} random variable is much smaller than 28.87, the conclusion about the homogeneity for the exponentiality with common parameters à and θ can be made with sufficient certainty.

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